Instructions: Try these problems from the textbook as practice (answers below).

Problems

13-22. Evaluate the triple integral.

13. $\iiint_E y\ dV$, where $E = \{(x, y, z) : 0 \le x \le 3,\ 0 \le y \le x,\ x - y \le z \le x + y\}$
15. $\iiint_E (1/x^3)\ dV$ where $E = \{(x, y, z) : 0 \le y \le 1, y \le x \le 1, 0 \le z \le xy\}$
17. $\iiint_E 6xy\ dV$ where E lies under the plane $z = 1 + x + y$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x},\ y = 0$ and $x = 1$
19. $\iiint_T y^2\ dV$ where T is the solid tetrahedron with vertices $(0, 0, 0), (2, 0, 0), (0, 2, 0)$ and $(0, 0, 2)$
21. $\iiint_E x\ dV$ where E is bounded by the paraboloid $x = 4y^2 + 4z^2$ and the plane $x = 4$

23-26. Use the triple integral to find the volume of the given solid.

23. The tetrahedron enclosed by the coordinate planes and the plane $2x + y + z = 4$
25. The solid enclosed by the cylinder $y = x^2$ and the planes $z = 0$ and $y + z = 1$

31. Sketch the solid whose volume is given by the iterated integral $\int_0^1 \int_0^{1-x} \int_0^{2-2z} dy\ dz\ dx$.

33-36. Express the integral $\iiint_E f(x, y, z)\ dV$ as an iterated integral in six different ways where E is the solid bounded by the given surfaces.

33. $y = 4 - x^2 - 4z^2$, $y = 0$
35. $y = x^2$, $z = 0$, $y + 2z = 4$

39. Write five other iterated integrals that are equal to the iterated integral $\int_0^1 \int_y^1 \int_0^y f(x, y, z)\ dz\ dx\ dy$.

41. Evaluate the triple integral $\iiint_C (4 + 5x^2yz^2)\ dV$ where C is the cylindrical region $x^2 + y^2 \le 4$, $-2 \le z \le 2$ using only geometric interpretation and symmetry.

Answers

13. $\dfrac{27}{2}$ 15. $\pi/8 - \dfrac{1}{3}$ 17. $\dfrac{65}{28}$ 19. $\dfrac{8}{15}$ 21. $\dfrac{16\pi}{3}$ 23. $\dfrac{16}{3}$ 25. $\dfrac{8}{15}$

33.

$$\begin{aligned}
&\int_{-2}^{2}\int_{0}^{4-x^2}\int_{-\sqrt{4-x^2-y}/2}^{\sqrt{4-x^2-y}/2} f(x,y,z)\ dz\ dy\ dx \\
&\quad= \int_{0}^{4}\int_{-\sqrt{4-y}}^{\sqrt{4-y}}\int_{-\sqrt{4-x^2-y}/2}^{\sqrt{4-x^2-y}/2} f(x,y,z)\ dz\ dx\ dy \\
&\quad= \int_{-1}^{1}\int_{0}^{4-4z^2}\int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x,y,z)\ dx\ dy\ dz \\
&\quad= \int_{0}^{4}\int_{-\sqrt{4-y}/2}^{\sqrt{4-y}/2}\int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x,y,z)\ dx\ dz\ dy \\
&\quad= \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{4-x^2-4z^2} f(x,y,z)\ dy\ dz\ dx \\
&\quad= \int_{-1}^{1}\int_{-\sqrt{4-z^2}}^{\sqrt{4-z^2}}\int_{0}^{4-x^2-4z^2} f(x,y,z)\ dy\ dx\ dz
\end{aligned}$$

35.

$$\begin{aligned}
&\int_{-2}^{2}\int_{x^2}^{4}\int_{0}^{2-y/2} f(x,y,z)\ dz\ dy\ dx \\
&\quad= \int_{0}^{4}\int_{-\sqrt{y}}^{\sqrt{y}}\int_{0}^{2-y/2} f(x,y,z)\ dz\ dx\ dy \\
&\quad= \int_{0}^{2}\int_{0}^{4-2z}\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y,z)\ dx\ dy\ dz \\
&\quad= \int_{0}^{4}\int_{0}^{2-y/2}\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y,z)\ dx\ dz\ dy \\
&\quad= \int_{-2}^{2}\int_{0}^{2-x^2/2}\int_{x^2}^{4-2z} f(x,y,z)\ dy\ dz\ dx \\
&\quad= \int_{0}^{2}\int_{-\sqrt{4-2z}}^{\sqrt{4-2z}}\int_{x^2}^{4-2z} f(x,y,z)\ dy\ dx\ dz
\end{aligned}$$

39.

$$\begin{aligned}
&\int_{0}^{1}\int_{0}^{x}\int_{0}^{y} f(x,y,z)\ dz\ dy\ dx \\
&\quad= \int_{0}^{1}\int_{z}^{1}\int_{y}^{1} f(x,y,z)\ dx\ dy\ dz \\
&\quad= \int_{0}^{1}\int_{0}^{y}\int_{y}^{1} f(x,y,z)\ dx\ dz\ dy \\
&\quad= \int_{0}^{1}\int_{0}^{x}\int_{z}^{x} f(x,y,z)\ dy\ dz\ dx \\
&\quad= \int_{0}^{1}\int_{z}^{1}\int_{z}^{x} f(x,y,z)\ dy\ dx\ dz
\end{aligned}$$

41. 64π